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"Falling into disuse": the rise and fall of Froebelian mathematical folding within British kindergartens

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ABSTRACT
This article aims to present the transformations as well as the decline of the Froebelian occupation of paper folding that took place in Great Britain between 1851 and the 1920s. Froebel's original intention was to transmit implicit mathematical knowledge to be learnt by means of folding several shapes. In contrast to his account of paper folding, which presented it as a way of understanding several abstract mathematical concepts, the manner in which this occupation was re-conceptualised in Great Britain tells a different story. After surveying the changes in the British conception of Froebelian ideas in the second half of the nineteenth century through the various published manuals, I then explore the period between 1892 – the opening of the Froebel Educational Institute – and 1924. Looking at the examination syllabi of the National Froebel Union, the examination papers, and their various appendices, as well as studying the approaches of the British mathematical community to folding at the end of the nineteenth century, the article will try to illuminate – via the inspection of the decline of mathematical education via folding in kindergartens – how formal work became discredited and eventually, in part, disappeared.

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Introduction: Froebel's conception of folding

This article explores how Froebelian folding – being one of the later activities introduced by Froebel within his array of gifts and occupations – was accepted, transformed, and eventually declined in Great Britain, starting from 1851 until the 1920s. Froebel himself considered paper folding as an activity whose *main* aim was to introduce and show geometrical and mathematical concepts and relations to children. However, this mathematical character, once considered – starting from the end of the nineteenth century – as mainly presenting objects in the real world or merely symmetrical forms, was seen as unnecessary and secondary. Indeed, Froebel's idea was to give children a series of objects and activities meant to reveal to the individual child fundamental forms and shapes. These objects and activities were called *gifts* and *occupations*, or in German "*Spielgaben*" and "*Beschäftigungsmittel*", as Froebel himself called them. They were devised and meant to be self-educating, in the sense that the child through the action of play, discovered their structures and laws. While

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gifts for Froebel included three-dimensional objects (for example, the ball, made of wood and thread, the cube and the divided cube, the cylinder) the *occupations* consisted of the following activities: cutting, gluing, folding, weaving, etc., working with various materials, not necessarily three-dimensional objects.

Froebel introduced paper folding *formally* only in 1850, i.e. at a later stage of his teaching. However his direct followers always took it to be an essential activity, indicating that it was present in Froebel's activities before 1850, as well as in the nurseries before Froebel's time. But for Froebel, paper folding was not just an activity for the sake of merely folding – that is, as a pleasurable activity. Froebel emphasised the mathematical nature of paper folding, prompting the understanding of geometrical as well as arithmetical concepts. But how was this mathematical character preserved, transformed, or neglected when Froebelian methods were integrated into the British education system? This article seeks to recognise the shifts within the conception of Froebelian folding in Great Britain, starting in 1851. While in the second half of the nineteenth century one can observe a growing acceptance of paper folding as a mathematical activity, in the first decades of the twentieth century a shift occurred. Reflecting a growing criticism of folding as an explicit mathematical procedure, especially in elementary schools, on the one hand, and on the technical nature of a few of the Froebelian occupations on the other, folding was hardly considered mathematically, was marginalised, and was slowly taken out of the syllabi of the learning programmes for teachers. But before dealing with these different conceptions, I will turn first to how Froebel himself formalised folding mathematically.

As mentioned above, it was only during 1850 that one finds Froebel explicitly treating paper folding systematically. Froebel, however, did deal with paper folding earlier. Around July 1845, Froebel writes to Leonhard Woepcke about the transformation of forms via paper folding:

> [a] further new and great, so entertaining, instructive and useful division of the occupations for the child is the *transformation* of the forms […] from flexible surfaces, from paper. The breaking and *folding* of different shapes and objects from one and the same square, or, what is the same: from several evenly sized squares.[1]

Froebel also writes to Berthold Auerbach in 1847, including a "folding box" in his description of the occupation materials.[2] Compared to these non-detailed letters, in 1850 Froebel wrote down his first elaborate remarks on the importance of paper folding in terms of the mathematical understanding of form, size, length, and relations between shapes.

[1]"Eine weitere neue und große, so unterhaltende als belehrende und nützliche Abtheilung der Kinderbeschäftigungen ist das Umwandeln der Formen und zwar […] aus biegsamen Flächen, aus Papier; das Brechen und *Falten* verschiedener Formen und Gegenstände aus einer und eben derselben Geviertfläche, oder was das Gleiche ist: aus mehreren gleich großen Geviertflächen." (cursive by M.F.). In: Friedrich Froebel to Leonhard Woepcke, in *Gesamtausgabe der Briefe Friedrich Fröbels*, Research Library for the History of Education, Berlin <http://bbf.dipf.de/editionen/froebel/fb1845-07-21-01.html>, July 1845 (Marienthal) v.21./28.7.1845 (Keilhau) (BN 699, Bl 1–8, 3 datierte Entwürfe fol. a) BN 699, Bl 1–4R; b) BN 699, Bl 5–6R; c) BN 699, Bl 7–8R; BN 58, Bl 1–24, undat. Abschrift 12 B 8° 48 p.) (accessed 1 May, 2017).

[2]Friedrich Froebel to Berthold Auerbach, in *Gesamtausgabe der Briefe Friedrich Fröbels*, Research Library for the History of Education, Berlin <http://bbf.dipf.de/editionen/froebel/fb1847-02-23-01.html>, February 1847 (BN 365, Bl 1–2, hier: 2–2R u 1–1R, undat. Entwurf 1 Bl 8° 2 S. auf 2–2R, mit undatierter Literaturliste/Spielmaterialien 1 Bl 8° 1 ½ p.) (accessed 1 May, 2017), 1.

Detour: Froebel and mathematics

In order to contextualise Froebelian methods of folding within his general conception of mathematics, it is essential to make a detour and to examine the way Froebel considered mathematics and its importance for young children. As is well known, Froebel worked between 1808 and 1810 at Johann Heinrich Pestalozzi's institute at Yverdon and was influenced by his methods. It was at Yverdon that Froebel obtained training in Pestalozzi's methods. Remember that Pestalozzi (1746–1827) had advanced education for all children, promoting Rousseau's ideas while preferring the use of tactile objects and the self-activity of the child on account of discipline and memorisation.[3] However, in 1811 Froebel left for Göttingen to study natural sciences, and then moved to Berlin in 1812, studying mineralogy under Christian Samuel Weiss (1780–1865). After military service, he worked at the mineralogical museum in Berlin, which influenced him deeply, as can be seen in his 1826 book *Die Menschenerziehung*: "you see the phenomena of nature not as in a dream […]. [Rather] nature is enduring; it surrounds you everywhere; it is solid, forming hard figures and resting on a crystal world".[4]

Although Froebel resigned from his position in 1816, it is clear that his educational philosophy and school pedagogy was deeply affected by his research on crystals and from his mathematical background. In *Die Menschenerziehung* he describes the law which governs the connections between man and nature, inner and outer, as a mathematical law: "What is mathematics according to its nature, its emergence, its effect? As an appearance of the inner and the outer world, it is equally affiliated to the human being and to nature."[5] This law is thus called the spherical law, through which all things are in connection, and this is to be seen most clearly through geometric forms, which are presented later with his gifts and occupations. The spherical law is a law that mediates the inner and the outer. This can already be seen in August 1811, when Froebel wrote down an initial formulation of the spherical law:

> [t]here is only one basic law throughout the whole universe. […] This law [is] the law of the + and – or the one of opposition. This law emerges from the centre towards all directions simultaneously or spherically. All that is, is subject to this spherical law.[6]

This mediation of opposites might resemble a Hegelian approach, or at least indicate the influence of German idealism, an influence which would later be heavily criticised within other European countries. I discuss this influence on Froebel briefly below. Additionally, it is important to note that this mediation between opposites was then conceptualised (also) in the form of paper folding, a theme I will deal with in the following section.

Another source of influence on Froebel's educational philosophy was his research on crystallography, and especially on the notion of transformation. As noted above, Froebel emphasised in his letter to Woepcke the "transformation of forms" emerging from paper folding. Indeed, one of the phenomena to be found in crystals that most influenced Froebel, on his arrival in Berlin, was "that even in these […] so called dead stones and masses, further

[3]For an analysis of Pestalozzi's conception of how mathematics should be taught to children, see the extensive analysis in Elmar-Bussen Wagemann, *Quadrat – Dreieck – Kugel* (Weinheim/Bergstrasse: Julius Beltz, 1957), 3–85.
[4]Friedrich Fröbel, *Die Menschenerziehung* (Keilhau/Leipzig: Wienbrack, 1826), 247.
[5]Ibid., 249.
[6]Erika Hoffmann and Reihhold Wächter, *Friedrich Fröbel. Ausgewählte Schriften. Briefe und Dokumente über Keilhau. Erster Versuch der sphärischen Erziehung*, vol. 5 (Stuttgart: Klett-Cotta, 1986), 355 (translation by M.F.).

developing activity and effectiveness still takes place. In the variety of form and design, I recognised a most variedly modified law of *development and formation*".[7] The transformation of forms – from one form to another, and not specifically the law governing their form – was a theme that fascinated Froebel, and held a special importance in his work. For Froebel, the principle of transformation was one of the basic principles that he adopted and adapted from crystallography.[8]

Taking this into consideration, the following question can be posed: did Froebel consider knowledge of mathematical truths and laws essential for the education of young children? As can be seen from examining the first three gifts, the answer is clearly positive. These gifts – balls, spheres, cylinders, cubes, and especially the divided cube introduced to the children – were not only amusements, but also and especially concepts of three-dimensional geometrical forms. As Norman Brosterman notes regarding the cube – which is divided into eight smaller cubes – while the children played with these cubes, "they learned form, size, position, and combination, as well as elementary mathematics and solid geometry".[9] These truths were learnt via the playful activities with the gifts and the occupations.

The question remains as to whether the educational goal of teaching mathematical notions to young children fit within what was common practice in kindergartens during the nineteenth century. As mentioned above, Froebel was influenced by Pestalozzi's approach to the education of children. As Elmar-Bussen Wagemann claims,[10] Froebel agrees with Pestalozzi that the child should deal already in kindergarten with counting concrete numbers, then abstracting them, eventually learning the abstract concept of number. When compared to Pestalozzi, however, Froebel goes one step further by introducing, already in *Die Menschenerziehung*, geometrical concepts such as various triangles, quadrangles etc.,[11] which are also eventually abstracted.[12] It should indeed be noted that when compared to Pestalozzi, Froebel was developing a novel method with respect to three-dimensional geometry. This can be clearly seen when one examines the relations between three-dimensional and one-dimensional geometry. According to Froebel, the research of forms should begin with three-dimensional objects, and only then should a passage to the investigation of two- and one-dimensional forms take place. Compared to his contemporaries Pestalozzi and Johann Friedrich Herbart, Froebel's method is indeed revolutionary.[13] This is equally true when Froebel's method is compared to one of the more accepted ways to learn geometry such as Euclid's *Elements*, which began from one-dimensional forms and only subsequently moved to treat three-dimensional forms. Moreover, Froebel also suggests that the child, and not only the adult, assumes the inner mathematical relations that govern nature.[14]

* * *

Keeping Froebel's conception of mathematical education for children in mind, I now turn to his elaborate analysis of folding in his short 1850 article entitled "Instructions for

[7]Wichard Lange, *Friedrich Fröbels gesammelte pädagogische Schriften. Erste Abteilung: Friedrich Fröbel in seiner Erziehung als Mensch und Pädagoge. Bd. 1: Aus Fröbels Leben und erstem Streben. Autobiographie und kleinere Schriften* (Berlin: Enslin, 1862), 111–12 (cursive by M.F.).

[8]See Wagemann, *Quadrat – Dreieck – Kugel*, 186–8.

[9]Norman Brosterman, *Inventing Kindergarten* (New York: Harry N. Abrams, 1997), 51.

[10]Wagemann, *Quadrat – Dreieck – Kugel*, 151.

[11]Ibid., 153.

[12]Ibid., 155. See Friedeich Fröbel, *Ausgewählte Schriften*, vol. 1, ed. Erika Hoffmann (Godesberg: Helmut Küpper, 1951), 61: "The forms and shapes of the real objects have [. . .] for the most part an extension in all directions in space." ["Die Formen und Gestalten der wirklichen Gegenstände haben [...] größtenteils Ausdehnung nach allen Richtungen im Raume."].

[13]Wagemann, *Quadrat – Dreieck – Kugel*, 158–9.

[14]Ibid., 220–1.

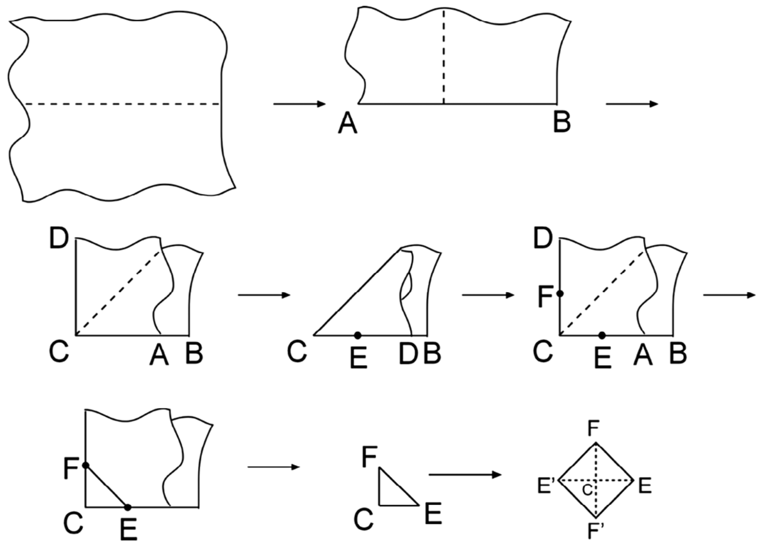

Figure 1. Depiction of Froebel's instructions for folding a square from an "unformed" piece of paper.
Note: Figure drawn by M.F.

Paper Folding [*Anleitung zum Papierfalten*]: A Fragment. A Developing-Educating and Entertaining-Instructing Children's Activity for Children from Five to Seven Years and Over, with the Extensive Participation of Adults".[15] This article is structured in two parts: the first is called "the guiding themes concerning the occupations in general, as a clear introduction into the particular"; the second "paper folding as an occupation, emanating from the quadratic surface or form".[16] One may assume that Froebel intended to continue developing this theme since the text ends with the following remark, that "here is now given the first basic form, from which the first principal forms necessarily should also be developed",[17] but none of his later writings dealt with this subject. What is therefore the "first basic form" suggested by the Froebelian occupation of paper folding? The form is, as expected, a square. Indeed, the second part deals with the various ways in which one can fold a square (*Geviert*), folding which not only exemplifies mathematical principles but an activity from which one also may induce several geometrical propositions and arithmetical concepts.[18]

In the second part, Froebel begins his description of paper folding with the creation of a square from *any piece of paper*, which might be of an irregular shape, but which is in any case without any tear. Froebel uses neither notation in this text nor draws any figure, giving only textual description. He begins by folding the piece of paper, creating a crease (*AB* in Figure 1) and then folds this crease onto itself, such that a second crease (*CD*) is formed,

[15]Friedrich Froebel, *Gesammelte pädagogische Schriften*, vol. 2, ed. Wichard Lange (Berlin: Enslin, 1874), 371–88. The title in German is: "Anleitung zum Papierfalten. Ein Bruchstück. Eine entwickelnd-erziehende und unterhaltend-belehrende Kinderbeschäftigung für Kinder von 5 bis 7 Jahren und darüber, unter eingehender Mitwirkung von leitenden Erwachsenen".

[16]"Leitender Faden bei den Beschäftigungsmitteln im Allgemeinen, als übersichtliche Einleitung in das Besondere", respectively "Das Papier-falten als Beschäftigungsmittel, von der Geviertfläche oder Geviertform ausgehend".

[17]Froebel, *Gesammelte pädagogische Schriften*, vol. 2, 388: "ist nun die erste Grundform gegeben, aus welcher sich nun auch die ersten Hauptformen mit Nothwendigkeit entwickeln".

[18]The first part offers a comprehensive published analysis of gifts and occupations. See Helmut Heiland, *Das Spielpädagogik Friedrich Fröbels* (Hildesheim/Zürich/New York: Georg Olms, 1998), 155.

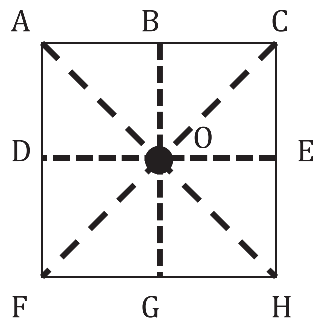

Figure 2. The basic diagram of Froebel, where *O* denotes the centre of the square. The diagonal creases are created, for example, by folding the edge *AF* on *FH* (creating *CF*). The vertical creases are created, for example, by folding *CH* on AF (creating *BG*).
Notes: Figure drawn by M.F. A similar drawing (without the notation) appears in Mary J. Lyschinska, *The Kindergarten Principle* (London: Isbister, 1880), 84.

perpendicular to the first. Froebel then folds the second crease on the first (*CD* on *CA*, as in Figure 1). He denotes now randomly a point *E* on *CA* and *CD* (now folded on each other) and unfolds the last fold. The point that was denoted as *E* now turns into two points, *F* on *CD* and *E* on *CA*, and by cutting along the line *EF*, the resulting, cut out, completely unfolded figure is a square, whose centre is *C* and whose vertices are *E, E', F,* and *F'*.

Froebel notes that "from […] the unformed continuous […] via three folding and three cuts the most regular and simplest form – the square – arises. […] [F]rom the unformed through pattern-following separation, the form proceeds, and here, in this particular case, the square".[19] Two of Froebel's observations already deserve emphasis here: firstly, that folding should not only be practical but also should be exact, otherwise when the edge *CD* falls only approximately on *CA*, the resulting form, after cutting along *EF*, would not be a square; secondly, Froebel stresses that folding is a technique which gives rise, by means of several transformations, to the simplest and the most regular form: the square. While in the first occupations the regular shapes were always already given (sphere, cube, cylinder), folding here is presented as a technique that enables the forming of shapes.

In the rest of his instructions, Froebel continues and takes now the square as given, guiding the reader through numerous phases, where the child folds the square along various lines and creases. Although I will not go over all of the steps, there are several important conclusions that Froebel does derive from these procedures that I would like to highlight while taking into consideration how he folds the square in Figure 2.

In step number ten (of Froebel's instructions), he notes that two different forms may have the same area. He then remarks that the area of the triangle *AFH* is half of the area of the original square. Later, he notes that the area of the rectangle *ABGF* is also half of the area of the original square. This leads him to declare, "half is the same as a half".[20] Two different forms may have the same size, the same area.

Froebel also considers shapes of the same form, for example the original square and the square *DOGF*. He concludes that "the same form does not necessitate the same size, or the

[19]Froebel, *Gesammelte pädagogische Schriften*, vol. 2, 376: "daß aus dem stetig Ungeformten, […] durch drei Brüche und drei Schnitte die gesetzmäßigste und einfachste Form, das Geviert, entsteht. […] aus dem Ungeformten durch gesetzmäßige Trennung das Geformte und hier, in diesem besonderen Falle, das Geviert hervorgeht".
[20]Ibid., 379: "Halbes ist […] gleich Halbem".

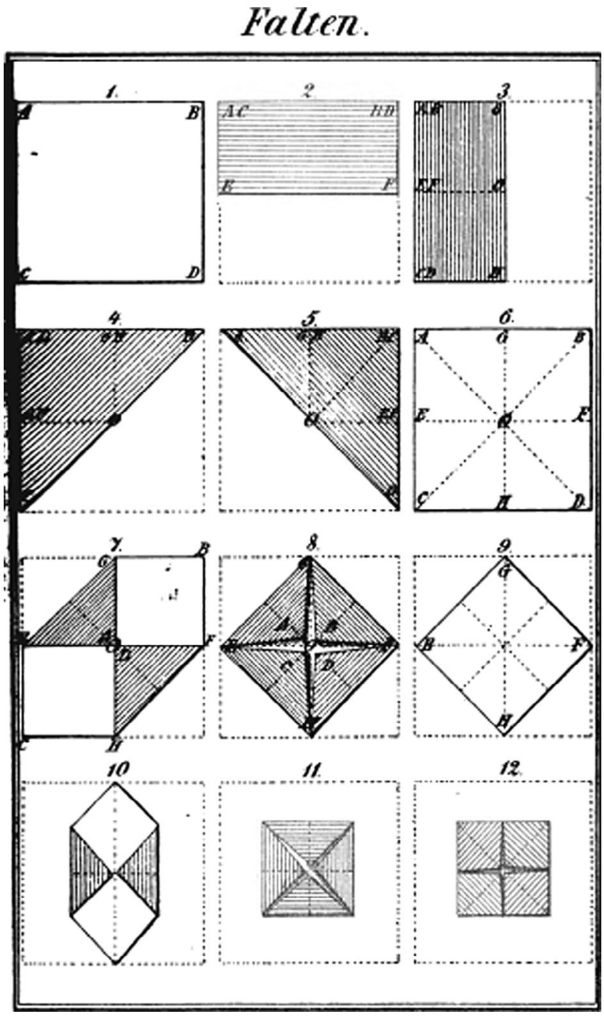

Figure 3. Froebel's process of folding (*Falten*) as depicted in 1874 by Hermann Goldammer. Compare also Figure 4.
Notes: Hermann Goldammer, *Fröbels Beschäftigungen für das vorschulplichtige Alter* (Berlin: Habel, 1874), plate 40.

size can be very different for the same form",[21] because the area of the square *DOGF* is a quarter of the area of the original square. Moreover, he remarks that the area of the triangle *FOH* is also a quarter of the original square. This leads him to say that a "quarter is the same as a quarter".[22] However, the next step is more complicated: Froebel folds along the lines *BD*, *DG*, *GE*, and *EB* (see Figure 2), thus obtaining a square, whose area is half of the original square (as in Figure 3 (9)). Taking the square presented in Figure 3 (9), Froebel folds it in such a way as to obtain the shape in Figure 3 (12). The area of the square obtained thus is a quarter of the area of the original square.[23]

[21] Ibid., 383: "gleiche Form bedingt nicht gleicht Größe, oder bei gleicher Form kann die Größe sehr verschieden sein".
[22] Ibid., 386: "Viertel ist gleich Viertel".
[23] Ibid., 387–8.

What can be inferred from Froebel's use of folding with regard to its mathematical treatment? Firstly, Froebel does not introduce any abstract concept of a number – or, to be more exact, of a fraction – but rather induces it from the relations between the various shapes. As outlined above, obviously the concept of fraction was already introduced in an implicit fashion with the divided cube, but this cube was *already* given as divided. With the folding of paper the child by contrast actively divides the form for himself. Indeed, Froebel's statements: "half is the same as a half" or a "quarter is the same as a quarter" indicate that the notion of the fraction as an abstract entity is derived from comparing different shapes, arriving at the conclusion that their size (half or quarter of the initial form) is equal. Explicitly, the fold enables the presentation of number as a relation: the area of the triangle *AFH* (in Figure 2) is half *of* the area of the original square.[24] The area as such, of any form described, is not measured at all, but only the relations between the areas. The emphasis on the pedagogical side that avoids abstract concepts is also to be noticed when Froebel writes about the truths of intuition (*Anschauungs-Wahrheit*),[25] which serve as a basis for truths by means of conclusions (*Wahrheitssätzen durch Schluß*),[26] i.e. through reasoning that is based on former proven theorems. The practical intuitive evidence suffices for Froebel as he hardly ever considers "the question of the validity of the mathematical […]; 'intuitive' evidence was enough for him in order to be able to speak about geometrical truths".[27]

Secondly, it is to be noted that Froebel uses only one form – the square; though he declares twice that an equilateral triangle (*Gedritt*) will be constructed through the folding of the square,[28] he does not construct it. There are several ways to make this, but one way is easily seen here: by looking at Figure 2, fold the base *FH* (when the point *F* stays fixed) such that the point *H* will be placed on *BG* and mark it by *M*; the triangle *MFH* will be equilateral.

The Froebelian insight that folding mediates between the formed and the unformed, and that it is an essential example of the transformation of forms, tempts one to conclude that this resembles a possible Hegelian conception where folding would be conceptualised as embodying the unity between two opposites – thus leading to a synthesis between them. While Froebel was indeed categorised as a "Hegelian" at the beginning of the twentieth century, this was merely the result of his first interpreters[29] and was in reality not at all the case. Although Froebel was influenced by German idealism, he was much more influenced by Schelling than by Hegel.[30] Irene Lilley notes that "it is […] futile to look in Froebel's theories for strict dialectical progression in Hegelian terms […]; his phraseology was not intended to be philosophically exact".[31] Froebel in fact denies that Hegel's dialectic was his inspiration, since he did not study his writings.[32]

[24]Ibid., 386–7.
[25]Ibid., 382, 383.
[26]Ibid., 382.
[27]Wagemann, *Quadrat – Dreieck – Kugel*, 252.
[28]Froebel, *Gesammelte pädagogische Schriften*, vol. 2, 375, 378.
[29]"Early interpreters […] intensified the idealist basis and dialectical pattern of Froebel's thought, and justified the Kindergarten methods and universal 'laws' of development in markedly Hegelian terms." In Irene Lilley, *Friedrich Froebel* (Cambridge: Cambridge University Press, 1967), 28–9. Recall that Froebel became familiar in 1802 with the writings of Schelling: *Von der Weltseele* [On the Soul of the World] and *Bruno oder über des natürliche und göttliche Prinzip der Dinge* [Bruno, or On the Natural and Divine Principle of Things]. Froebel "explained the obvious conflict and tension which creative growth involves in Schelling's terms of an underlying identity of opposites which, having a common ground, can be reconciled" (Lilley, *Friedrich Froebel*, 10).
[30]Ibid., 8.
[31]Ibid., 11.
[32]See Barbara Bratty, *Preschool Education in America: The Culture of Young Children from the Colonial Era to the Present* (New Haven/London: Yale University Press, 1997), 42.

What, however, is emphasised by Froebel, when he discusses folding, is the transformation of forms. These folding transformations enabled not only the visualisation of mathematical truths but also the abstraction of the concept of a number, in a way that is not based on counting. Thus, for example, the insight that half of a square can be obtained via various folded forms reveals that the concept of number is independent of form and size. While the divided cube also exemplifies this, folded paper initiates this understanding by means of the haptic activity of the child who folds the material.

Considering these novel methods of introducing mathematics via playful activities and especially via folding already in kindergartens,[33] how were the Froebelian methods transferred and changed after his death? The following section will deal with how mathematical folding was accepted in Great Britain.

A partial acceptance in Great Britain: "laying the foundation" for geometrical understanding

To what extent were Froebel's ideas regarding the teaching of mathematics in general, and by means of folding in particular, integrated into the educational system of Great Britain? In this section I will concentrate on several books and manuals, which followed and developed Froebel's approach in Great Britain. The spread of the Froebelian movement in Europe was in part due to Bertha von Marenholtz-Bülow who began to lecture and exhibit Froebel's writings, pedagogical methods, occupations, and gifts outside Germany. Thus, for example, she came to Paris in 1855, where she stayed for two years, and integrated the Froebelian system into the French kindergarten.[34] Another main motor in the dissemination of Froebelian theories, especially in Great Britain, were German immigrants who moved to Great Britain after it was forbidden in 1851 by the Prussian government to educate the children in kindergartens (a ban that officially ended in 1861). Thus, in 1851 Johannes and Bertha Ronge opened the first Froebelian kindergarten in London and in 1873 the Froebel Society was founded in Manchester – a year later a branch was founded in London as well. Towards the end of the nineteenth century kindergartens were integrated into some public schools in Great Britain yet Froebel's ideas faced revision; they thus failed to achieve universal recognition.[35]

Taking into account the partial success of the integration of Froebel's methods, one question remains: how was the Froebelian mathematical conception of folding integrated in the

[33]One of the reasons why Froebel's techniques of paper folding were well received in Germany was perhaps due to their resemblance to the form of baptismal letters. Beginning in the eighteenth century in Germany, these letters were folded in a form called "Blintz" or "Doppel-Blintz". The end result of such folding was a folded square either in the form depicted in Figure 3 (8) or 3 (11). It is reasonable to think that such shapes may indeed have inspired Froebel, as his own baptismal letter was also folded. See, for example, John Sallas, *Gefaltete Schönheit. Die Kunst des Serviettenbrechens* (Wien: Freiburg i. Breisgau, 2010), 94–5.

[34]For the different ways Froebel's pedagogy was accepted and integrated in Germany, France, and Italy, see Michael Friedman, *A History of Folding in Mathematics: Mathematizing the Margins* (Basel: Birkhäuser, 2018), section 4.2.1.3.

[35]Several papers have dealt with the reception, acceptance, and transformations of Froebelian pedagogy in Great Britain. See, for example: Kevin J. Brehony, "The Froebel Movement and State Schooling, 1880–1914: A Study in Educational Ideology" (PhD thesis, Open University, Milton Keynes, 1987); Kevin J. Brehony, "English Revisionists, Fröbelians and Schooling of the Urban Poor," in *Practical Visionaries: Women, Education and Social Progress, 1790–1930*, ed. Mary Hilton and Pam Hirsch (London/New York: Routledge, 2000), 183–99; Kristen Dombkowski, "Kindergarten Teacher Training in England and the United States 1850–1918," *History of Education* 31, no. 5 (2002): 475–89; Jane Read, "Froebelian Women: Networking to Promote Professional Status and Educational Change in the Nineteenth Century," *History of Education* 32, no. 1 (2003); Jane Read, "Free Play with Froebel: Use and Abuse of Progressive Pedagogy in London's Infant Schools, 1870–c.1904," *Paedagogica Historica* 42, no. 3 (2006): 17–33.

curriculum of kindergartens in Great Britain? As we will see, in most of the textbooks, the abstract mathematical terms and concepts – which were to be derived from the truths of intuition, being exemplified by folding – appeared in a sporadic way, and were later ignored or even declared as unsuitable for young children. The manuscripts, written during the second half of the nineteenth century, that did mention the mathematical aspects of Froebel's paper folding, were conceived later as too technical, limiting the imagination of the child.

While reviewing the manuals, I will mainly follow the division given by Kevin J. Brehony of the years 1851 to 1911 concerning the reception of Froebel's ideas in Great Britian:[36] the establishment of private kindergartens (1851–70); the beginning of state elementary schooling (1870–96); and revisionism (from 1896); though, as noted in the introduction, this paper will take 1924 as a concluding year.

Surveying the manuals chronologically, I begin with Bertha Ronge (1818–63), who represents the first period of acceptance of Froebel's ideas through private kindergartens. Ronge was an educator and an activist, supporting and advancing children's and women's education in Germany and England. She was largely responsible for the spreading of the kindergarten movement in England during the 1850s and afterwards,[37] opening several kindergartens. In 1855 Bertha Ronge and her husband, Johannes, published their manual *A Practical Guide to the English Kindergarten*, which appeared in several subsequent editions. After giving an ample description of the various main gifts, a small section is dedicated to folding called "Paper Folding: With an Undivided Square Form". Most of the section deals the "[t]he forms of recognition which […] are especially important and practical as a means of conveying an idea of many important truths in geometry".[38] However, the geometrical truths that are described are simplified in comparison to the ideas Froebel presented. Although following the way Froebel folded his square piece of paper, the geometrical truths presented only deal with the superposition of figures and the fact that the area of a triangle is half of an area of a rectangle with the same height and base. Moreover, the truths presented are only specific to right-angled triangles.[39]

The Education Act was declared in Great Britain in 1870, which led the Froebelian movement to seek integration in various ways into the public system. This integration happened rather slowly, if at all. A possible explanation was the foreign origin of the occupations and gifts. Whereas some elements were integrated, several Froebelians demanded the acceptance of the entire system.[40] I now review the manuscripts and manuals that deal explicitly with paper folding, that were published during this second period, spanning from 1870 until the end of the nineteenth century.

Mary Gurney, being one of the driving forces of the 1871 foundation of the Women's Education Union, was a one of the "long-term activists in the Froebel movement and joined the [Froebel] Society in its first year of existence".[41] Writing several manuals on how to use and practice the Froebelian occupations, she translated and abridged August Köhler's *Praxis*

[36]Kevin J. Brehony, "The Kindergarten in England 1851–1918," in *Kindergartens and Cultures: The Global Diffusion of an Idea*, ed. Roberta Wollons (New Haven: Yale University Press, 2000), 59–86, 60.

[37]See: Brehony, "The Kindergarten in England 1851–1918," 61–4; and Manfred Berger, "Ronge, Bertha," *Biographisch-Bibliographisches Kirchenlexikon*, vol. XX (Nordhausen: T. Bautz 2002), columns 1225–1229.

[38]Bertha Ronge and Johannes Ronge, *A Practical Guide to the English Kindergarten* (London: Hodson, 1855), 48.

[39]For example, ibid.: "By these foldings it will be clearly seen that the base of each triangle is longer than either of its sides, and that the two acute angles of each triangle are together equal to one right angle".

[40]See Brehony, "The Kindergarten in England 1851–1918," 70.

[41]Read, "Froebelian Women," 27.

des Kindergartens, published in several parts, starting from the mid-1870s. The third edition of *Kindergarten Practice: Part II, Froebel's Plane Surface* was published in 1877. In this book, paper folding is numerated as Gift number 8.[42] Gurney mentions that:

> Froebel taught only one kind of Paper Folding – with squares, and applied only one rule in folding, i.e. folding the corners; the author of this work has added [a] new rule for folding squares, i.e. folding the sides, and for advanced pupils has introduced the folding of rectangles and of […] hexagons and octagons.[43]

This statement is incorrect (Froebel did not only fold "the corners"), as canbe seen from Froebel's short article "Instructions for Paper Folding", and it indicates that Gurney might not have had access to Froebel's writing about this subject.

Gurney also mentions that folding affords "good exercise […] for [the child's] fingers and hands […]. He also gains some of his earlier ideas in mathematics from the study of parts and angles". However, examining the exercises that Gurney provides, half of them aim "for the representation of real objects",[44] while for the other half, folding presents geometrical shapes. Gurney does not include Froebel's initial construction of the square from any piece of paper and it is not clear how the "earlier ideas in mathematics" are actually explained to the child. Thus, for example, the last set of exercises starts with an equilateral triangle "taken as a foundation",[45] which was certainly a novelty compared to Froebel, but it is not explained how this triangle is actually obtained in the first place.

The 1880 book by Mary Lyschinska (1849–1937), *The Kindergarten Principle*, reflects a more mathematical approach regarding paper folding in kindergarten. It was under Lyschinska's direction as the instructor of kindergartens in the London School Board that a wide range of the gifts and occupations were introduced, and among them, paper folding. As Jane Read notes, "[j]ust two years after her appointment [in 1879] Lyschinska introduced a 'Form and Number' syllabus that used Froebel's kindergarten materials – sticks, beads and rings – for counting and cardboard pieces for forming capital letters".[46] Focusing on folding, Lyschinska however remarks that it had already a marginal place within the gifts:

> Only a few of the less well-known occupations, therefore, have been chosen [to be presented in the chapter], and these have been handled with a view to furnishing teachers with some practical illustrations of the way in which these occupations are helpful in developing ideas of form, colour, and number in children […]. We group the exercises conveniently in the following way: Group A, Familiar Objects. […] Group B, Symmetrical Forms. Group C, Geometrical and Numerical Exercises.[47]

Lyschinska emphasises that "[i]n the geometrical group", the children

> distinguish the corners from the sides of a square, […] they know by experience that the sides are as long as one another, and they do not learn the name "square" or "oblong" until they know a good many of the properties of these several figures.[48]

In contrast to Gurney's instructions, Lyschinska points out explicitly that "with the geometrical ideas to be conveyed, the difference between the 'edge' and 'corner' may be conveyed

[42]Mary Gurney, *Kindergarten Practice: Part II, Froebel's Plane Surface* (London: George Philip & Son, 1877), 9.
[43]Ibid.
[44]Ibid., 10.
[45]Ibid., 13.
[46]Read, "Free Play with Froebel," 313.
[47]Lyschinska, *The Kindergarten Principle*, 64–5.
[48]Ibid., 75.

simply by asking a child to point to one and to the other"[49] Reflecting her "Form and Number" syllabus, she also explains how to introduce the notion of a fraction to the child, echoing the Froebelian ideas. Thus, for example, when presenting the notion of "fourth", she folds a square into four smaller squares (i.e. folding along *BG* and *DE* as in Figure 2) and asks "when four small pieces, all the same size, make one large piece, we give the small pieces a name. Do you know it? (Fourths)".[50]

A similar spirit, coupling folding and geometrical as well as mathematical knowledge, is to be found in the 1892 book by Maria Kraus-Boelté and John Kraus *The Kindergarten Guide Number Nine: the 7th and the 8th Occupations*. Fifty pages are dedicated to paper folding, and the authors state at the beginning of the treatment that paper folding's

> elementary simplicity fits it for being made an occupation for the little ones. Almost every thing that has been gained, by previous Occupations, in *mathematical* intuition, is found again in Paper-Folding, which is a real compendium of elementary mathematics. Surfaces, lines, corners, angle and figures of various kinds are seen.

However, they immediately add,

> this is not the prominent point to which to direct the child's attention. It is the great variety of representation of things which we see around us […] and Forms of Symmetry, wherein lies the chief source of interest to the children.[51]

Despite these remarks, which might undermine the importance of paper folding as a geometrical tool, the mathematical account given regarding folding is quite extensive.[52] The authors explain meticulously how the initial steps of Froebelian folding should be done, following Froebel's own instructions, followed by the drawings that appear in Figure 4.

Maria Kraus-Boelté and John Kraus state that geometrical paper-folding "teach[es] almost every geometrical form".[53] The exercises follow the Froebelian tradition: "Simple mathematical truths are thus early demonstrated, such as: 'Halves of the same thing are equal'; and 'the sum of the parts is equal to the whole.'"[54] The emphasis in the exercises is given however not to concept of the number or of the fraction, but mainly to the discovery and visualisation of geometrical forms (trapezoid, pentagon, hexagon, octagon). Indeed, while the authors remark that "paper-folding also offers instruction in numbers, fractions and angles for the paper is folded and refolded, divided into halves, quarters, eighths, thirds, etc., [and that] [i]t is of great benefit to the child to acquire all this mathematical knowledge",[55] they hardly explain what the concept of the fraction transmitted through folding is. The increasing level of complexity of the exercises, ending with complex forms as in Figure 5, also raises the question as to whether the child can indeed "develop all the forms himself".[56]

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[49]Ibid., 77.
[50]Ibid., 81.
[51]Maria Kraus-Boelté and John Kraus, *The Kindergarten Guide Number Nine: the 7th and the 8th Occupations* (London: Myers, 1892), 252.
[52]Ibid., 284–96.
[53]Ibid., 284.
[54]Ibid., 286.
[55]Ibid., 301.
[56]Ibid.

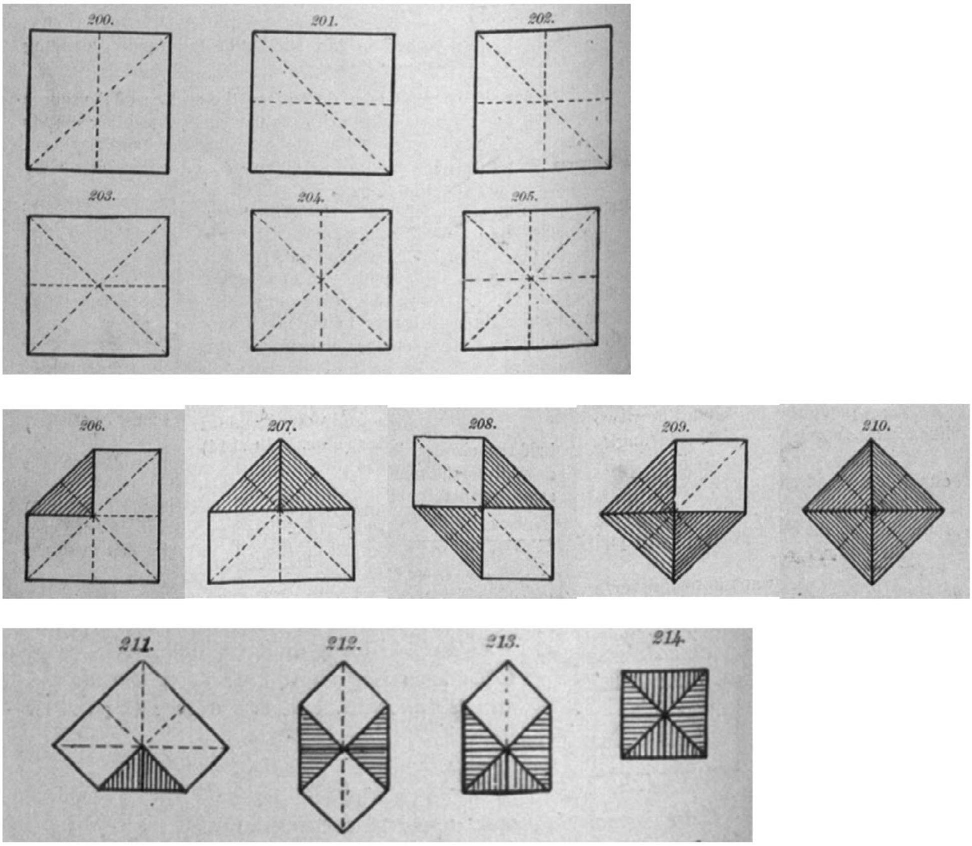

Figure 4. Figures 200–14 from Kraus-Boelté and Kraus, *The Kindergarten Guide Number Nine*, 288–90, presenting the different stages of the Froebelian folding.

Turning to Eleonore Heerwart (1835–1911), from the 1860s onwards she was one of the main figures in the Froebelian movement in England.[57] In her 1897 book *Froebel's Theory and Practice* Heerwart emphasises the importance of children finding geometrical symmetrical forms by themselves, and that folding is not only be used as an amusement.[58] Heerwart however only gives a summary of Froebel's "Instructions for Paper Folding"[59] and lists of "later additions to the paper folding", noting among others the regular pentagon, hexagon, and octagon. She states that paper folding is more useful in teaching forms than numbers and notes that through "lines, angles, and the geometrical plane figures appear before the child, and the accuracy of these adds greatly to the beauty of paper folding. While the younger children need not learn all the geometrical names, they produce forms nevertheless".[60] In Heerwart's book

[57]For a more extensive study of Heerwart, see: Berger, Manfred, "Heerwart, Eleonore." in *Lebenswege in Thüringen. Dritte Sammlung*, ed. Felicitas Marwinski (Weimar: Historische Kommission für Thüringen e. V., 2006), 147–51; Rosemarie Boldt, "Einige Gedanken über das Fröbelverständnis von Eleonore Heerwart," in *Friedrich Fröbel. Aspekte international vergleichender Historiographie*, ed. Helmut Heiland, Karl Neumann, and Michael Gebel (Weinheim: Beltz, 1999), 160–9; Rosmarie Boldt, "Eleonore Heerwart über die Gaben und Beschäftigungen Friedrich Fröbels," in *Fröbels Pädagogik. Verstehen, interpretieren, weiterführen*, ed. Helmut Heiland and Karl Neumann (Würzburg: Könighausen & Neumann 2003), 104–19.

[58]Eleonore Heerwart, *Fröbel's Theory and Practice* (London: Charles & Dible, 1897), 56.

[59]Ibid., 57.

[60]Ibid.

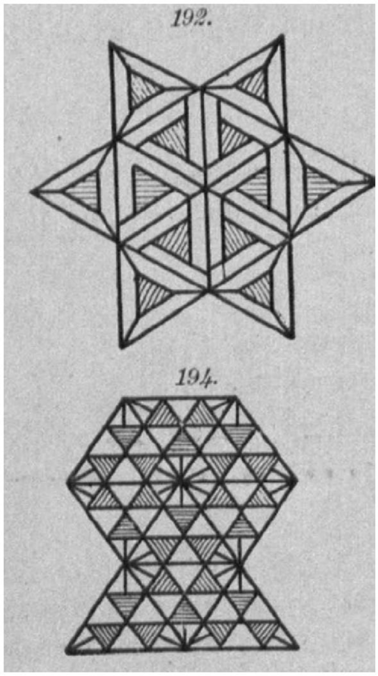

Figure 5. Figures 192 and 194 from Kraus-Boelté and Kraus, *The Kindergarten Guide Number Nine*, 284, presenting folded examples of *forms of symmetry* and *forms of life*.

Course of Paper-Folding,[61] a more comprehensive treatment of the matter is to be found. Mentioning at first that folding as an activity was present in nurseries in order to amuse children,[62] even before Froebel presented it as an occupation, Heerwart then distinguishes the older activity of folding from the Froebelian one. Not only do the Froebelian activities present "symmetrical or geometrical forms"[63] and show the importance of "follow[ing]" the instruction in "a logical order", but moreover, in "[p]aper-folding the previous knowledge of typical forms, cubes, etc., is represented in a new manner, the newness consisting in the child's producing on his own account".[64]

Although Heerwart describes Froebel's initial steps for folding as he described them in "Anleitung zum Papierfalten", her book *Course of Paper-Folding* also contains "geometrical forms [which] are later additions".[65] Among them one can find the equilateral triangle,[66] where she remarks, in line with Froebel, that this form is "not so natural, and we shall see […] that it is a derived form".[67] According to Heerwart, folding occupies an important place while teaching the concept of form: "Form reigns supreme, and indeed Paper-folding is an excellent preparation for drawing and geometry; it is easier to fold a line and an

[61]Eleonore Heerwart, *Course of Paper-Folding*: *One of Froebel's Occupations for Children, at Home and in the Kindergarten* (London: Charles & Dible, 1895).
[62]Ibid., 4. Gurney also notes this in: Gurney, *Kindergarten Practice: Part II, Froebel's Plane Surface*, 9.
[63]Heerwart, *Course of Paper-Folding*, 5.
[64]Ibid.
[65]Ibid., 11.
[66]Ibid., plate V. Heerwart remarks that, regarding this plate, all of the forms "were added after Froebel's time" (ibid., 15).
[67]Ibid., 5.

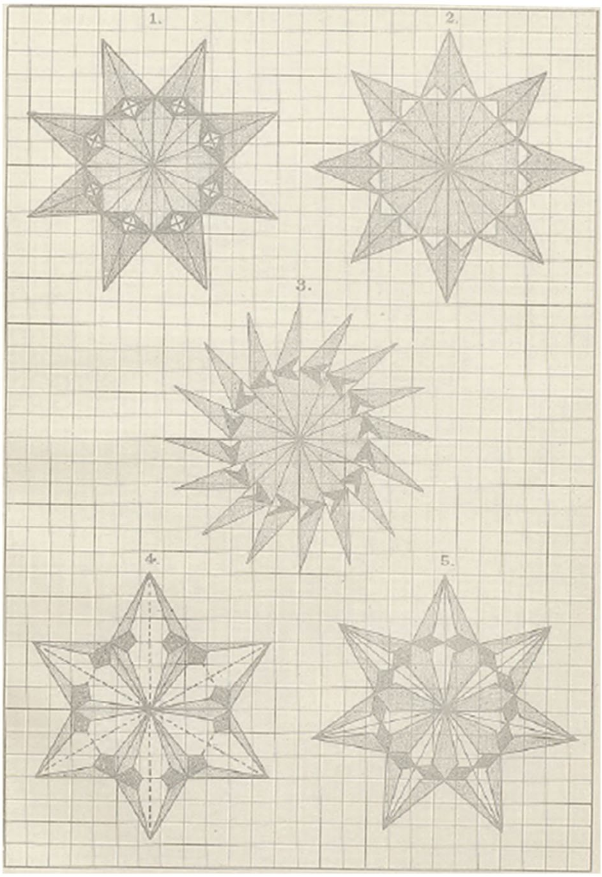

Figure 6. The emphasis on highly symmetrical forms that should be folded is clear in Heerwart's book *Course on Paper Folding*. Regarding these folding diagram (at plate XXIX), presented in the figure, she remarks that while "it would be laborious to produce the acute angle at 51 $\frac{3}{7}$ for the centre and 128 $\frac{4}{7}$ for the obtuse angle of the heptagon, it is quite easy by the folding of the rhombus as shown in the design". However, how the different rhombi are folded in practice is not explained.
Note: Heerwart, *Course of Paper-Folding*, 20.

angle than to draw it, therefore *folding* should *precede* the *drawing*."[68] This remark reflects the growing trend that avoids presenting arithmetical truths or concepts via folding, and rather concentrates on the geometrical forms and propositions that can be shown. It also presents alternative methods for the learning of geometry, in addition to drawing with the help of compass and straight edge, which are the usual instruments for teaching Euclidean geometry. And while Kate Douglas Wiggin and Nora Archibald Smith comment in 1902 that "paper folding […] lays the foundation, as does nothing else, for the acquiring of the fundamental principles of geometry",[69] it is clear that during the last decade of the nineteenth century a growing emphasis was put on the more symmetrical, and one might even say, technical hand work of folding (see Figure 6). The way that fractions might be introduced

[68]Ibid., 11.
[69]Kate Douglas Wiggin and Nora Archibald Smith, *Froebel's Occupations* (London: Gay and Bird, 1902), 225.

through folding was slowly marginalised in favour of a growing preference of showing geometrical, mostly symmetrical forms, as well as folded shapes that represent real objects.

By the close of the nineteenth century a tacit ignorance towards the notion of transformation and change, as exemplified by folding, took place. Although none of the authors surveyed above explicitly opposed these ideas, their absence should be noted: the final form was emphasised more than its transformation. The wish to avoid an emphasis on Froebel's philosophical ideas might have been a reason why the folding exercises and their description were revised (instead of, for example, being completely ignored). Indeed, these conceptions were already criticised in France in Pauline Kergomard's book *L'éducation maternelle dans l'école* of 1886[70] and, as I will show below, this critique can also be found in Great Britain.

A partial rejection: separation from Euclidean geometry and falling into disuse

Starting from 1896 one can note the beginning of the third period regarding the reception and reformulation of Froebel's ideas, characterised by the founding of free kindergartens,[71] under the influence of, among others, the ideas of John Dewey.[72] At the beginning of the twentieth century, the above manuscripts and manuals were criticised as obsolete; they were considered as having only a "superficial understanding of Froebel's educational philosophy",[73] which may indeed have been the cause of the very reformulation of his ideas. This change in the conception of Froebelian ideas can be seen at the institutional level. In 1887 the National Froebel Union (NFU) was founded as a joint examining institution, offering then official certificates in teacher training. In 1892, the Froebel Educational Institute, aiming at providing training for educators, opened in West Kensington, London, with Emilie Michaelis as principal. The Froebel College was also founded in 1892 as a teacher training college. Two courses were offered through the NFU: the first, elementary certificate, was especially for training educators for nurseries and kindergartens; the second, the higher certificate, was "intended for Teachers who undertake the full charge of a Kindergarten",[74] when the material learnt there was definitely meant to be taught thereafter to children going to elementary school. As follows, one can observe the change in the status of the Froebelian gifts and occupations seen in the syllabi and the exams given as part of these two courses over the years. While the gifts and occupations, including folding, were a part of the students' curriculum, as I will show, starting from the early twentieth century, they became secondary and were eventually partially removed from various teaching programmes.

Examining the syllabi of both the course for the elementary certificate and the higher certificate throughout the 1890s, one sees that both courses emphasised the importance of the practical knowledge of the "Kindergarten gifts and occupations".[75] "[C]andidates

[70]Even if Kergomard's study intended to adopt several parts of Froebel's folding exercises, most likely the activities that dealt with objects that resembled given objects in the world, she nevertheless objected strongly to "the invasion of the German spirit" that the reception of folding represented. It was thus that she claimed that the abstract, "philosophical", "spiritual", "mathematical" parts of Froebel's folding exercises should not be included in the curriculum of pre-school children. See Pauline Kergomard, *L'éducation maternelle dans l'école* (Paris: Hachette, 1886), 122–3.

[71]Brehony, "The Kindergarten in England 1851–1918," 75.

[72]Cf. Brehony, "English Revisionists, Fröbelians and Schooling of the Urban Poor".

[73]Read, "Free Play with Froebel," 321.

[74]National Froebel Union, *Examination Results and Reports of Examiners* (London: Office of the NFU, 1892), inner cover.

[75]National Froebel Union, *Syllabus of the Examination of the Joint Examination Board* (London: Office of the NFU, 1890), 6.

will be expected to show practical knowledge"[76] of the gifts and occupations, and specific instructions were given as to which forms should be folded. In the first appendix to the syllabus, called "Notes as to requirements in occupations", the first section is devoted to paper folding; for the elementary certificate it is suggested to teach a "[c]ourse fully worked out from the ground form, produced by a square with corners folded to the centre"; the higher certificate demanded "two symmetrical courses derived from any ground form, and a geometrical course worked out from the square".[77] This appendix appeared in all of the syllabi published in the 1890s.

One of the questions in the written exam for the higher certificate in 1892 was "Which of the Kindergarten gifts and occupations specifically lend themselves to the teaching of Geometry?" while another asked "What do you consider the educational value of the occupation of paper-folding?"[78] The section "gifts and occupations" in the exam for the elementary certificate of 1898 contains three questions (out of ten) that deal with paper folding. One of them connects paper folding and geometry directly: "Problem in Paper-Folding and Paper-Cutting. (a) from one given square show how to obtain: (i.) An equilateral triangle. (ii.) Two equal isosceles triangles."[79] However, already in the 1890s the place of paper folding for teaching geometry was somehow limited: Geometry, as presented in the exams for the higher certificate, was mainly meant to be taught via books I and II of Euclid's *Elements*.[80] Indeed, paper folding was considered in the examiners' reports as "practical geometry", while geometry as presented in Euclid's *Elements* was named simply as "geometry".[81] What this reveals is that Euclid's *Elements* had a more important place when it came to teaching the child the validity of geometrical truths than any form of practical (haptic) activity could evidence. Whereas the former could instruct, the latter could only show. Nevertheless, students themselves actively practised paper folding, and, in the teachers' handwork albums from the 1890s, to be found, for example, in the archives of the Froebel College at Roehampton, paper folding is indeed included.[82] Moreover, not only are the folded forms that are found in these albums the same as those found in the manuals surveyed above, but paper folding was actively practised by both genders in the Froebelian kindergartens during the 1900s, as can be seen in the photo in Figure 7.

However, at the turn of the century, the status of the Froebelian gifts and occupations, and hence of folding, began to change. Indeed, for the progressive Froebelians, eyes "were to be safeguarded from fine and exact paper folding [...] these were to be replaced by brushwork and free drawing. Free play [...] took the place of organised kindergarten games".[83] Elsie Murray considered this problematic approach in 1903. Born in 1861 in Scotland, she began to learn about Froebel's ideas in 1893 at the Maria Grey College. Rather than sticking to

[76]Ibid.
[77]Ibid., 15.
[78]National Froebel Union, *Examination Papers* (London: Office of the NFU, 1892), 54.
[79]National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1898), 86.
[80]Ibid., 79.
[81]National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1902), 8–9.
[82]For example, see the following albums: Album of Fanny Paul, Object paper folding, FACS/9/1/77, 1892, The Froebel Archive for Childhood Studies, University of Roehampton, London, England; Album of Charlotte Robina Muckle coursework, Kindergarten Occupations FACS/9/1/71, 1890s, The Froebel Archive for Childhood Studies, University of Roehampton, London, England.
[83]Phyllis Woodham-Smith, "History of the Froebel Movement in England," in *Friedrich Froebel and English Education*, ed. Evelyn Mary Lawrence (New York: Philosophical Library, 1953), 34–94, 88–9.

Figure 7. Children engaging with paper folding at Colet Gardens Demonstration School. The Colet Gardens Demonstration Kindergarten and School were opened in 1895 and were staffed with former students of the Froebel Educational Institute.
Notes: Colet Gardens Demonstration School, Paperfolding, FACS/15/6/1, c1900, The Froebel Archive for Childhood Studies, Archives and Special Collections, University of Roehampton, London, England.

orthodoxy and under the influence of Dewey (see below), she reinterpreted Froebel's principles, rejecting the theoretical ideas that were supposed to be transferred by means of the gifts. This is to be seen in her 1903 article "That Symmetrical Paper Folding and Symmetrical Work with Gifts are a Waste of Time for both Students and Children".[84]

In this article she declares,

> on the face of it […] these things are a waste of time. […] Can we put this value [of the occupations] into simple words […] so that the ordinary student, who does not understand the terms of Hegelian philosophy […] will cease to complain?[85]

While she does admit that every symmetrical work gives a certain amount of "aesthetical training", she adds that the occupations "demand the accuracy of a machine".[86] Symmetrical paper folding does not express the ideas of a child. Moreover, "they have nothing to do with the life of the child".[87] Aside from symmetry, other mathematical aspects are not even mentioned and there is not a single drawing of what could be regarded as "proper" paper

[84]Elsie R. Murray, "That Symmetrical Paper Folding and Symmetrical Work with Gifts are a Waste of Time for both Students and Children," *Child Life* V, no. 17 (1903): 14–8. Cf. also Woodham-Smith, "History of the Froebel Movement in England," 91–2.
[85]Murray, "That Symmetrical Paper Folding," 15.
[86]Ibid., 16.
[87]Ibid., 18.

folding; the only advantage of paper folding is attributed to the enjoyment of the child due to the work done with her or his own hands.[88]

What Read indeed suggests is that, under the influence of Dewey, "Froebelians re-imagined their practice and articulated a revisionist interpretation of Froebel which made his pedagogy fit for purpose in the new scientifically oriented twentieth-century world".[89] This is to be seen in Murray's critique above, where paper folding was considered to be what reflects a nineteenth-century "Hegelian philosophy" (as Murray stated above), which is imposed on children already in kindergarten. As we saw above, however, such Froebelian ideas, re-conceptualised from a Hegelian perspective (for example, regarding the transformation of forms), were already given less emphasis from the 1870s onwards.

What ought to be emphasised is that this "revisionist interpretation of Froebel" was mainly consequent to Dewey's revision of Froebel. Brehony claims that many "Froebelians […] claimed that Dewey had played the role of liberator and that his critique and reworking of Froebel's themes had contributed significantly to the crumbling of the old Froebelian orthodoxies".[90] Murray describes Dewey as "one of the few important educational writers who do justice to Froebel as a pioneer".[91] Following Dewey, Murray advised that the symbolism of the Froebelian occupations ought to be rejected in order to concentrate on the values the occupations provided for social training.[92] Murray, together with Maria Findlay and Henrietta Brown Smith, attacked the Froebelian kindergarten occupations. Insofar as they also had an institutional presence in the training of teachers in colleges, they successfully marginalised these occupations. Thus, for example, Maria Findlay, who in 1901 became a member of the governing body of the NFU, held the opinion that "occupations most suited to satisfying these needs were not […] those of manual training […] but, 'cooking, weaving and constructing'".[93] It is also possible that folding was rejected on the basis that it did not contribute to the Deweyian "co-operative and mutually helpful living",[94] as Murray put it. Taking this into account, it is not surprising that paper folding activities were transformed in the first decade of the twentieth century into cardboard modelling, which I discuss in detail below. This shift, along with the abandonment of highly symmetrical paper folding exercises, – due to the parallel shift from organised and teacher-led activities to free play – also reflected Dewey's discursive strategies on how young children learnt and ought to be taught. Consequently, these symmetrical teacher-guided shapes were then perceived as useless for the "social training" of the child.[95] Furthermore, the objection to and rejection

[88]See also Read, "Free Play with Froebel," 321–2.

[89]Jane Read, "Freeing the Child: Froebelians and the Transformation of Learning through Play, Self-Activity and Project Work in English State Junior School Classrooms, 1917–1952," in *Kindergarten Narratives on Froebelian Education: International Perspectives*, ed. Helen May, Kristen Nawrotzki, and Larry Prochner (London & New York: Bloomsbury, 2017), 139.

[90]Kevin J. Brehony, "An 'Undeniable' and 'Disastrous' Influence? Dewey and English Education (1895–1939)," *Oxford Review of Education* 23, no. 4 (1997): 427–55, 433.

[91]Elsie R. Murray, *Froebel As a Pioneer in Modern Psychology* (London: G. Philip, 1914), 6.

[92]Brehony, "An 'Undeniable' and 'Disastrous' Influence?," 437. See Murray's summary of Dewey's revision of Froebel's pedagogy: Murray, *Froebel As a Pioneer in Modern Psychology*, 7. According to Murray's conception of Dewey, "these individual […] activities [e.g. the Froebelian occupations] are organized and directed through the uses made of them in keeping up the co-operative living".

[93]Brehony, "An 'Undeniable' and 'Disastrous' Influence?," 438, 436–9.

[94]Murray, *Froebel As a Pioneer in Modern Psychology*, 7.

[95]Thus, for example, Murray mentions a teacher who let the children fold during the last ten minutes of the lesson. This may indicate that folding possibly served as a "time-filler" before the end of a lesson: Murray, "That Symmetrical Paper Folding," 18).

of folding, understood as an activity that possibly enabled the discovery of mathematical truths, was not only to be found within kindergarten.

As indicated above, while examining the exams for the higher certificate, Euclid's *Elements* formed the basis of the geometry taught in the schools of Great Britain in the nineteenth century;[96] however, not only was an alternative way of teaching geometry suggested via the Froebelian methods, but also several mathematicians expressed their discontent concerning this way of teaching. Dionysius Lardner, an Irish scientific writer best known for editing the *Cabinet Cyclopædia*,[97] was one of them.[98] In 1840 Lardner published his *Treatise on Geometry and its Application in the Arts*, which used folding as a legitimate method of proof and offered another way of acquiring geometrical knowledge by extending geometrical practice.[99] Two other mathematicians who saw folding as a legitimate tool for mathematical education in schools in Great Britain were Richard P. Wright and Olaus Henrici. In Wright's 1868 book *Elements of Plane Geometry*, folding is used several times and is also suggested, for example, as an inference method for proving the congruency of polygons – a successful folding of one shape over the other results in this congruency.[100] Olaus Henrici's 1879 book *Elementary Geometry: Congruent Figures* is another example of the usage of folding: Henrici rejected the rote learning of Euclid and considered folding a method of logical reasoning. When Henrici examines the bisection of angles and segments, he emphasises that by folding – either one ray of an angle on the other or one end of the segment on the other – one bisects either the angle or the segment.[101] But these approaches have not gone uncriticised: while Murray criticised Froebelian folding as being too mechanical and too philosophical, Henrici and Wright's approaches were criticised for being too childish, non-scientific, and cumbersome, and also for being too wordy. In 1885, Charles L. Dodgson (more popularly known as Lewis Carroll) criticised Henrici's book in the second edition of his *Euclid and His Modern Rivals*.[102] He compared the Euclidean proof and the Henrician proof of the proposition: "The greater side of a Triangle is opposite to the greater angle",[103] when Henrici uses

96Euclid's *Elements* was used as a textbook for learning geometry, almost without change, in Great Britain until the end of the nineteenth century. Regarding the debate on Euclid's method and the rival manuals that were proposed in Britain at that time, see: Amirouche Moktefi, "Geometry. The Euclid Debate," in *Mathematics in Victorian Britain*, ed. Raymond Flood, Adrian Rice, and Robin Wilson (Oxford: Oxford University Press, 2011), 527–42.

97On Lardner's life, see: Anna L. Martin, *Villain of Steam: A Life of Dionysius Lardner* (Carlow: Tyndall Scientific, 2015); and Jo N. Hays, "The Rise and Fall of Dionysius Lardner," *Annals of Science* 38 (1981).

98Dionysius Lardner, *The First Six Books of the Elements of Euclid* (London: John Taylor, 1828), ix: "Two thousand years have now rolled away since Euclid's *Elements* were first used […]; but still the 'Elements' themselves have been invariably preferred." See Friedman, *A History of Folding in Mathematics*, section 3.3, for an elaborate survey of the role of folding in the mathematical education in elementary schools in Great Britain during the nineteenth century.

99For example, Lardner bisects segments and angles and constructs right angles using folding; see Dionysius Lardner, *A Treatise on Geometry and its Application to the Arts, The Cabinet Cyclopaedia* (London: Longman et al., 1840), 32, 77, 18. See also Alice Jenkins, *Space and the "March of Mind": Literature and the Physical Sciences in Britain, 1815–1850* (Oxford: Oxford University Press, 2007), 158–61, regarding Lardner's view of the teaching of geometry.

100Richard P. Wright, *The Elements of Plane Geometry* (London: Longman et al., 1868), 75, 161.

101Olaus Henrici, *On Congruent Figures* (London: Longmans, Green and Co., 1879), 56. In addition, in India in 1893, the mathematician Tandalam Sundara Row published his book *Geometric Exercises in Paper Folding*: see Tandalam Sundara Row, *Geometric Exercises in Paper Folding* (Madras: Addison, 1893). The book was influenced by the British re-conceptualisation of the Froebelian methods of paper folding as well as from Henrici's usage of folding; Row refers to paper folding as kindergarten gift number 8 (i), and to Henrici's book *Elements of Plane Geometry* (64). Though the work of Row is directly influenced by Froebel, an analysis of his research and the influence of Row's book is beyond the scope of this paper; see: Michael Friedman, "Two Beginnings of Geometry and Folding: Hermann Wiener and Sundara Row," in *Journal of the British Society for the History of Mathematics* 31, no. 1 (2016): 52–68; and Friedman, *A History of Folding in Mathematics*, section 4.2.2.2.

102The first edition did not survey Henrici's book.

103Charles L. Dodgson, *Euclid and His Modern Rivals*, 2nd ed. (London: Macmillan, 1885), 94–5.

folding to prove it. Dodgson then remarked that Henrici's proof is too long.[104] Dodgson's explicit remarks with respect to folding are also very critical. While surveying Wright's book he remarked that "[a]ll that about folding and re-folding the paper is more like a child's book of puzzles than a scientific treatise. I should be very sorry to be the school-boy who is expected to learn this precious demonstration!"[105] Dodgson's judgement here symbolises how folding may have easily been degraded to mere demonstration, being similar to the degradation of Froebelian folding into merely the folding of forms which represent objects in the real world. These objections, which prompted, at the beginning of the twentieth century, the consolidation of the status of the Euclidean methods as the dominant way to teach geometry, are also to be seen in the syllabi and exams of the NFU: while in the exams of the higher certificate, Euclid's *Elements* and methods are often mentioned, a shift took place with respect to how gifts and occupations were regarded. What this indicates is that different didactic approaches (such as the Euclidean one) were preferred in teaching children mathematical truths. As far as the Froebelian occupations were concerned, they were criticised as unsuited to the needs of children (this was the opinion expressed by the revisionists). Hence the instruction of young children in mathematical truths was hardly accomplished by means of the (now revised) Froebelian occupations.

Two publications mark this shift: the first implicitly; the second explicitly. The implicit one is the 1905 book by Grace and William Young entitled *The First Book of Geometry*,[106] a text whose goal was to teach geometry to children using only paper folding techniques. The book does not mention Froebel at all, and it is not known whether the Youngs were aware of his methods. However, they do note that "paper-folding (without any didactic object or result) is a favourite amusement at the Kindergarten".[107] While the Youngs present numerous mathematical exercises done only with folding, exercises that are more advanced than those presented via Froebelian folding (see Figure 8), they eventually emphasise the advantages of axiomatic reasoning. According to the Youngs, the axiomatic approach of Euclid demands "verbal demonstrations" and "verbal reasoning",[108] while the *proofs* by paper folding demand only that the child operate with and fold paper. The distinction between the axiomatic approach and the paper folding operations become clearer when the Youngs inspect the logical reasoning behind the two practices. For example, after showing how to bisect a segment AB at a point M, by folding A on B, they take another segment PQ of equal length to AB, posit it on AB, where the point O on PQ corresponds to the point M of AB. At this point it is asked whether $PO = OQ$: "[i]t looks like it, but how do we know it for certain? It is because of our axiom – Things which are equal to the same are equal to one another".[109] Indeed, the Youngs imply the necessity of the axiomatic base, supporting logically what folding might show: while folding might show for the private cases the

[104]Ibid., 95.
[105]Ibid., 180. This critique appeared in the first edition (1879) of Dodgson's book.
[106]Grace Chisholm Young and William Henry Young, *The First Book of Geometry* (New York: Chelsea Publishing Company, 1905).
[107]Ibid., v. The Youngs also criticise Row's book *Geometric Exercises in Paper Folding* (see note 101 above), indicating "[t]he book of Sundara Row […] has little to recommend it. It is too difficult for a child, and too infantile for a grown person".
[108]Ibid., ix–x. The Youngs refer to Euclid explicitly (ix).
[109]Ibid., 57.

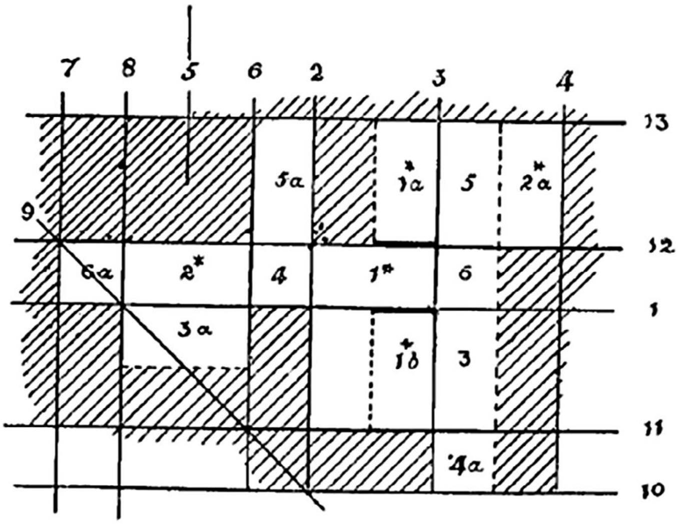

Figure 8. One of the more complicated folding exercises to be found at the Youngs' *The First Book of Geometry* for folding a box.
Note: Young and Young, *The First Book of Geometry*, 113.

validity of geometrical propositions, axioms and logic show it for every case.[110] Eventually, folding-based geometry, although more intuitive, is presented as a procedure to visualise the logical procedure already done (see Figure 8 from the Youngs' book of complicated folding exercises).

The publications of the NFU explicitly mark the above-mentioned shift. In 1905 the examination topic changed from the original "Gifts and Occupations" to "Gifts and Educational Handwork", followed by "Educational Handwork" in the following year. In 1906, in the syllabus for the elementary certificate, paper folding is mentioned under "special education handwork", when it is classified as "constructive" and as "preparatory to Cardboard Modelling".[111] The "educational handwork" part in the exam for the elementary certificate does not contain any question on paper folding, though a question on it does appear in the equivalent part in the exam for the higher certificate.[112] The appendix, which dealt with paper folding, is no longer added to the syllabus. A similar pattern is to be found in the "Regulations for the Award of Certificates" from 1909. Folding is presented as "constructive" under "special educational handwork" and it is to be done with – but also without – a ruler.[113] As for an example of such construction, which would be done without a ruler, this can be

[110]Ibid., 57–8: "here we have given all our reasons without leaving anything out; this is called *giving a proof*. Sometimes when something 'looks like it,' it is only because we have not drawn or folded quite accurately, so it is very necessary to be able to give a proof. When you are able to give your reasons, […] this is called *Logic*. […] Geometry will help you to learn logic. Now test by folding that PQ really is bisected at O".

[111]National Froebel Union, *Syllabus of the Examinations for the Lower Form and Kindergarten Mistresses* (London: Office of the NFU, 1906), 5.

[112]National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1906), 124, 140.

[113]National Froebel Union, *Regulations for the Award of Certificates* (London: Office of the NFU, 1909), 22.

found in the construction of regular polygons.[114] The fact that it was acknowledged that folding enables the construction of geometrical objects *without the use of a ruler* shows that folding techniques might have been recognised at the beginning of the 1900s as what enables mathematical reasoning (and not just modelling) independent of Euclidean instruments (in this case, ruler and compass). In other words, the child could readily understand that two (or all) of the edges are equal to each other precisely by means of her/his construction, i.e. by folding one edge on the other. Nevertheless, it must be noted that this understanding came too late and consequently was all but ignored.

Indeed, the 1909 examination papers for the elementary certificate do not mention paper folding under the part on "educational handwork", as well as the part on "geometry" in the examination papers for the higher certificate – though this part, along with questions regarding geometrical proofs, does require the construction of models of three-dimensional objects from cardboard. The "Regulations for the Award of Certificates" from 1913 omit paper folding almost completely from "educational handwork", mentioning it only generally as what "satisfies [the] love [of the children] for design" by "making objects interesting" to them.[115] Finally, in 1916, "education handwork" transforms into merely "handwork", and the corresponding part of the exam for the higher certificate asks the following question: "For what reasons do you think the original 'Kindergarten occupations' (paper folding, mat-plating, tablet-laying, &c.) have fallen into disuse? Discuss the probability of their ever being revived."[116] A similar question appears in the 1920 exam of the higher certificate of the "handwork" section: "For what reasons have some of the old Kindergarten occupations gone out of favour?"[117] And while in 1924 a "handwork diploma" could be obtained, none of the proposed subjects is paper folding.[118]

Conclusion

Froebel's original intention, that the child will experience and learn geometric forms as well as arithmetic relations and concepts via folding as what also essentially embodied the transformation of the formed and the unformed, was usually accepted in British Froebelian kindergartens during the second half of the nineteenth century. This certainly reflected the other attempts to teach geometry at elementary schools that did not involve Euclid's *Elements* in Great Britain at that time. Even within the British Froebelian movement, however, there was a growing tendency to prefer the symmetrical forms. This led to a conception of Froebelian paper folding that conceived of it as *either* too technical (or too tedious), *or* too "Hegelian", even if the latter already underwent considerable marginalisation from the 1870s onwards. The manuals, which presented folding as an essential activity and that were once considered highly significant, were regarded at the beginning of the twentieth century

[114] This construction was one of the folding exercises that can be found in the teachers' albums at the archives of the Froebel College at Roehampton. See, for example, Album of Charlotte Robina Muckle coursework, Kindergarten Occupations FACS/9/1/71, 1890s, The Froebel Archive for Childhood Studies, University of Roehampton, London, England. No evidence of using a marked ruler is found when looking at these folding exercises.

[115] National Froebel Union, *Regulations for the Award of Certificates* (London: Office of the NFU, 1913), 21.

[116] National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1916), 53.

[117] National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1920), 41.

[118] National Froebel Union, *Reports of Examiners, Examination Papers and Examination Results* (London: Office of the NFU, 1924), 13.

as "imposing adult-led activities instead of allowing children free scope".[119] The shift with respect to how the Froebelian gifts and occupations were regarded in turn affected how folding was later conceived. Seen as what *only* presents symmetrical forms for the sake of symmetry, this activity could be dispensed with. Folding was then presented as "constructive", enabling the construction of models, and at the same time, there is no mention of what enables the discovery of mathematical truths. Hence, it is no wonder that in the 1920s folding had "gone out of favour".

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[119]Read, "Free Play with Froebel," 321.